B.Tech.
(SEM. I) ODD SEMESTER THEORY EXAMINATION 2012-13
MATHEMATICS—I

Time : 3 Hours

Total Marks : 100

SECTION—A

1. All parts for this question are compulsory : (2×10=20)

(a) Find the 8th derivative of \( x^2e^x \).

(b) If \( x^2 = au + bv, y^2 = au - bv \), then find \( \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial u} \).

(c) Find the stationary points of
\[ f(x, y) = 5x^2 + 10y^2 + 12xy - 4x - 6y + 1. \]

(d) If \( x = u(1 + v), y = v (1 + u) \), then find the Jacobian of \( u, v \) with respect to \( x, y \).

(e) Reduce the matrix \[
\begin{bmatrix}
1 & 1 & 1 \\
3 & 1 & 1 \\
\end{bmatrix}
\]
into normal form.

(f) Prove that the matrix \( A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix} \) is unitary.

(g) Evaluate \( \int_0^1 \int_0^x e^y \, dx \, dy \).

(h) Evaluate \( \Gamma(-3/2) \).

(i) Find the value of \( m \) if \( \bar{F} = mx\hat{i} - 5y\hat{j} + 2z\hat{k} \) is a solenoidal vector.
(j) Find the unit normal at the surface \( z = x^2 + y^2 \) at the point 
\((1, 2, 5)\).

SECTION — B

2. Attempt any three parts of the following: \((3 \times 10 = 30)\)

(a) If \( y = \left( x + \sqrt{1 + x^2} \right)^m \), then find the \( n \)th derivative of \( y \) at 
\( x = 0 \).

(b) Find the maximum and minimum distance of the point 
\((1, 2, -1)\) from the sphere \( x^2 + y^2 + z^2 = 24 \).

(c) Find the eigen values and eigen vectors of the following matrix:

\[
\begin{bmatrix}
3 & 10 & 5 \\
-2 & -3 & -4 \\
3 & 5 & 7
\end{bmatrix}
\]

(d) Evaluate \( \iiint_V (ax^2 + by^2 + cz^2) \, dx \, dy \, dz \) where \( V \) is the 
region bounded by \( x^2 + y^2 + z^2 \leq 1 \).

(e) Verify Gauss’s divergence theorem for the function 
\( \vec{F} = x^2 \hat{i} + z \hat{j} + yz \hat{k} \) over unit cube.

SECTION — C

Attempt any two parts from each question of this section. All 
questions are compulsory. \([(2 \times 5) \times 5 = 50)]

3. (a) State and prove Euler’s theorem for homogeneous 
functions.

(b) Expand \( f(x, y) = e^x \tan^{-1} y \) in powers of \((x - 1)\) and \((y - 1)\) 
upto two terms of degree 2.
(c) If \( z = f(x, y) \) where \( x = e^u \cos v, y = e^u \sin v \), prove that
\[
\left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 = e^{-2u} \left[ \left( \frac{\partial f}{\partial u} \right)^2 + \left( \frac{\partial f}{\partial v} \right)^2 \right].
\]

4. (a) If \( x + y + z = u, y + z = uv, z = uvw \), then find \( \frac{\partial (x, y, z)}{\partial (u, v, w)} \).

(b) The two sides of a triangle are measured as 50 cm and 70 cm, and the angle between them is 30°. If there are possible errors of 0.5% in the measurement of the sides and 0.5 degree in that of the angle, find the maximum approximate percentage error in measuring the area of the triangle.

(c) Show that \( u = y + z, v = x + 2z^2, w = x - 4yz - 2y^2 \) are not independent. Find the relation between them.

5. (a) Test the consistency and hence, solve the following set of equations:
\[
10y + 3z = 0,
3x + 3y + 2z = 1,
2x - 3y - z = 5,
x + 2y = 4.
\]

(b) Using elementary transformations, find the rank of the following matrix:
\[
A = \begin{bmatrix}
-2 & -1 & 3 & -1 \\
1 & 2 & -3 & -1 \\
1 & 0 & 1 & 1 \\
0 & 1 & 1 & -1
\end{bmatrix}
\]

(c) Examine the following vectors for linearly dependent and find the relation between them, if possible:
\[
X_1 = (1, 1, -1, 1), X_2 = (1, -1, 2, -1), X_3 = (3, 1, 0, 1).
\]
6. (a) Prove that: \[ \sqrt{\pi} \Gamma(2n) = 2^{2n-1} \Gamma(n) \Gamma\left(n + \frac{1}{2}\right), \]
where n is not a negative integer or zero.

(b) Change the order of integration and hence evaluate
\[ \int_0^\infty \int_0^y ye^{-y^2/x} \, dx \, dy. \]

(c) Find the area of the region occupied by the curves \( y^2 = x \)
and \( y^2 = 4 - x \).

7. (a) Show that the vector field \( \vec{F} = yz \hat{i} + (zx + 1) \hat{j} + xy \hat{k} \) is conservative. Find its scalar potential. Also find the work done by \( \vec{F} \) in moving a particle from \((1, 0, 0)\) to \((2, 1, 4)\).

(b) Prove that:
\[ \text{Curl}(\vec{F} \times \vec{G}) = \nabla \times (\vec{F} \times \vec{G}) = \nabla \times \vec{F} \times \nabla \vec{G} - \vec{F} \times (\nabla \times \vec{G}). \]

(c) If \( \text{div} [f(r) \vec{r}] = 0 \), where \( \vec{r} \) is the position vector of a point \((x, y, z)\), then find \( f(r) \).