B. Tech.

(Semester-I) Theory Examination, 2012-13

ENGINEERING MATHEMATICS-I

Time : 3 Hours] [Total Marks : 100

Note : Attempt questions from each Section as per
instructions. The symbols have their usual meaning.

Section-A

Attempt all parts of this question. Each part
carries 2 marks. 2×10=20

1. (a) If \( y = x^2 \exp(2x) \), determine \((y_n)_0\).

(b) Find the radius of curvature for the curve
\( s = \log(\tan \psi + \sec \psi) + \tan \psi \sec \psi \), where \( \psi \)
is the angle which the tangent at any point to
the curve makes with the \( x \)-axis.

(c) If \( u(x, y) = \left( \sqrt{x} + \sqrt{y} \right)^5 \), find the value of
\[
\left( x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} \right).
\]
(d) The formula, $V = kr^4$, says that the volume $V$ of fluid flowing through a small pipe or tube in a unit of time at a fixed pressure is a constant times the fourth power of the tube’s radius $r$.

How will a 10% increase in $r$ affect $V$?

(e) Use Beta function to evaluate:

$$\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}} \, dx$$

(f) Changing the order of integration in the double integral:

$$I = \int_0^8 \int_{\pi/4}^{2} f(x, y) \, dx \, dy$$

leads to

$$I = \int_r^s \int_p^q f(x, y) \, dy \, dx$$

say,

What is $p$?

(g) If $\vec{F} = \frac{r}{r^3}$, find $\text{curl} \vec{F}$.

(h) Using Green’s theorem, evaluate the integral:

$$\oint_C (xy \, dy - y^2 \, dx),$$

where $C$ is the square cut from the first quadrant by the lines $x = 1$, $y = 1$. 

(2)
(i) If $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ are the characteristic roots of the $n$-square matrix $A$ and $k$ is a scalar, prove that the characteristic roots of $[A - kI]$ are $\alpha_1 - k, \alpha_2 - k, \alpha_3 - k, \ldots, \alpha_n - k$.

(ii) Explain the working rule to find the inverse of a matrix $A$ by elementary row or column transformations.

Section-B

Attempt any three parts of this question. Each part carries 10 marks. $10 \times 3 = 30$

2. (a) Find the values of $a$ and $b$ such that the expansion of $\log(1 + x) - \frac{x(1 + ax)}{(1 + bx)}$ in ascending powers of $x$ begins with the term $x^4$ and hence find this term.

(b) Locate the stationary points of:

$$x^4 + y^4 - 2x^2 + 4xy - 2y^2$$

and determine their nature.

(c) Evaluate:

$$\int_R \int (x - y)^4 \cdot \exp(x + y) \, dx \, dy,$$
where $R$ is the square in the $xy$-plane with vertices at $(1, 0), (2, 1), (1, 2)$ and $(0, 1)$.

(d) Verify the Gauss divergence theorem for:

$$\vec{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$$

taken over the rectangular parallelepiped $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$.

(e) Diagonalize the following matrix $A$:

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}.$$

Section-C

Attempt all questions of this Section. Attempt any two parts from each question. Each question carries 10 marks. $10 \times 5 = 50$

3. (a) If $y = \sin[\log(x^2 + 2x + 1)]$, prove that:

$$(x+1)^2y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2 + 4)y_n = 0.$$ 

(b) Trace the curve $y = x(x^2 - 1)$.

(c) Show that the radii of curvature of the curve $y^2 = \frac{x^2(a+x)}{(a-x)}$ at the origin are $\pm a\sqrt{2}$. 

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4. (a) The rate of flow $Q$ of water per second over the sharp-edged notch of length $l$, the height of the general level of the water above the bottom of the notch being $h$, is given by the formula

$$Q = c \left( l - \frac{h}{5} \right) h^{3/2},$$

where $c$ is a constant. Show that for small error $\delta h$ in the measurement of $h$, the error $\delta Q$ in $Q$ is:

$$\frac{1}{2} c (3l - h) h^{1/2} \cdot \delta h.$$

(b) Show that the envelope of the family of parabolas

$$\left( \frac{x}{a} \right)^{1/2} + \left( \frac{y}{b} \right)^{1/2} = 1,$$

under the condition $ab = c^2$ ($a$, $b$ and $c$ are constants), is a hyperbola whose asymptotes coincides the axes.

(c) Expand $f(x, y) = y^x$ about $(1, 1)$ up to second degree terms and hence evaluate $(1.02)^{1.03}$. 

(5)
5. (a) Find the mass of the solid bounded by the ellipsoid \( \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \) and the coordinate planes where the density at any point \( P(x, y, z) \) is \( kxyz \), where \( k \) is a constant.

(b) Prove that:

\[
\int_0^m \frac{\sqrt{\pi} (2m)}{(2)^{2m-1}} \left( m + \frac{1}{2} \right)
\]

(c) Evaluate:

\[
\int_0^\infty \int_0^x x \exp\left(-\frac{x^2}{y}\right) \, dx \, dy.
\]

6. (a) A particle moves along a plane curve such that its linear velocity is perpendicular to the radius vector. Show that the path of the particle is a circle.

(b) Find the directional derivative of \( v^2 \), where \( \vec{v} = xy^2 \hat{i} + zy^2 \hat{j} + xz^2 \hat{k} \) at the point \((2, 0, 3)\) in the direction of the outward normal to the sphere \( x^2 + y^2 + z^2 = 14 \) at the point \((3, 2, 1)\).
(c) Evaluate \( \int_C \vec{F} \cdot d\vec{r} \) along the curve \( x^2 + y^2 = 1, \ z = 1 \) in the positive direction from \((0, 1, 1)\) to \((1, 0, 1)\), where:

\[
\vec{F} = (yz + 2x)\hat{i} + xz\hat{j} + (xy + 2z)\hat{k}.
\]

7. (a) Find the characteristic equation of the matrix:

\[
A = \begin{bmatrix}
2 & 1 & 1 \\
0 & 1 & 0 \\
1 & 1 & 2
\end{bmatrix}
\]

and hence find the matrix represented by

\[
A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I,
\]

where \( I = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \).

(b) Investigate for what values of \( \lambda, \mu \) the simultaneous equations:

\[
x + y + z = 6, \ x + 2y + 3z = 10, \ x + 2y + \lambda z = \mu
\]

have (i) no solution, (ii) a unique solution (iii) an infinite number of solutions.
(c) If $N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$ is a matrix, then show that $(I-N)(I+N)^{-1}$ is unitary matrix, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.